Canadian
Mathematics
Competition

An activity of the Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Tuesday, February 19, 2008

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1. Do not open the Contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name, city/town, and province in the box in the upper left corner.

5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**

6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.

7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

8. Diagrams are *not* drawn to scale. They are intended as aids only.

9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site,
http://www.cemc.uwaterloo.ca.

Part A: Each correct answer is worth 5.

1. What is the value of $3^2 - 2^2 + 1^2$?

    (A) 8 **(B)** -2 **(C)** 10 **(D)** -5 **(E)** 6

2. $\dfrac{\sqrt{25-16}}{\sqrt{25}-\sqrt{16}}$ is equal to

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

3. Which of the following numbers is closest to 1?

    (A) $\frac{3}{4}$ **(B)** 1.2 **(C)** 0.81 **(D)** $1\frac{1}{3}$ **(E)** $\frac{7}{10}$

4. A bag contains 5 red, 6 green, 7 yellow, and 8 blue jelly beans. A jelly bean is selected
    at random. What is the probability that it is blue?

    (A) $\frac{5}{26}$ **(B)** $\frac{3}{13}$ **(C)** $\frac{7}{26}$ **(D)** $\frac{4}{13}$ **(E)** $\frac{6}{13}$

5. The 5-digit number $52\,28\square$ is a multiple of 6. Which digit is represented by $\square$?

    (A) 0 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 8

6. If $\dfrac{40}{x} - 1 = 19$, then x is equal to

    (A) -1 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** -2

7. In the diagram, what is the perimeter of polygon
    $PQRST$?

    (A) 24 **(B)** 23 **(C)** 25

    (D) 26 **(E)** 27
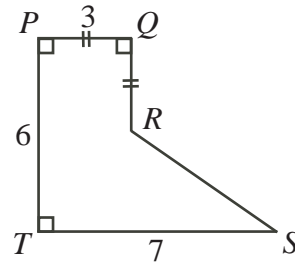
8. In the diagram, PRT and QRS are straight lines.
    What is the value of x?

    (A) 45 **(B)** 50 **(C)** 55

    (D) 60 **(E)** 65
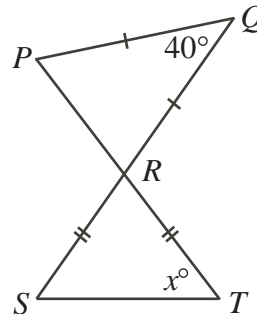
9. If $a = 7$ and $b = 13$, the number of even positive integers less than ab is

    (A) $\dfrac{ab-1}{2}$ **(B)** $\dfrac{ab}{2}$ **(C)** $ab - 1$ **(D)** $\dfrac{a+b}{4}$ **(E)** $(a-1)(b-1)$

10. Vivian's cell phone bill includes the graph showing her cell phone use for the month. She is charged

   - $20 per month, plus
   - 10¢ per minute for daytime calls, plus
   - 5¢ per minute for evening calls after the first 200 evening minutes. (The first 200 evening minutes are free.)
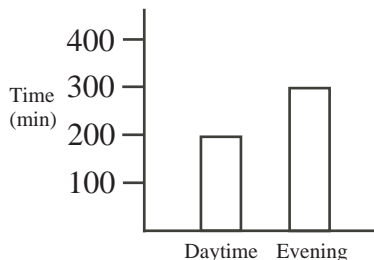
   What is her total cell phone bill for the month shown?

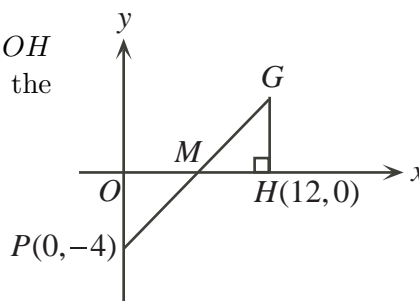   (A) $25   (B) $40   (C) $45
   (D) $70   (E) $75

Part B: Each correct answer is worth 6.

11. Lex has $2.65. He has only dimes (worth $0.10 each) and quarters (worth $0.25 each). If Lex has more quarters than dimes, how many coins does he have in total?
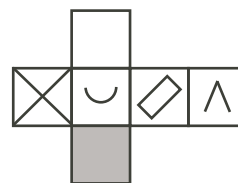
    (A) 12   (B) 13   (C) 16   (D) 19   (E) 22

12. The line from G through the midpoint M of OH intersects the y-axis at $P(0, -4)$. What are the coordinates of G?

    (A) $(12, 3)$ (B) $(12, 7)$ (C) $(12, 5)$
    (D) $(12, 6)$ (E) $(12, 4)$

13. The diagram shows a piece of cardboard that can be folded to make a cube. The cardboard has designs on one side only. Which one of the following cubes can be made from this cardboard?

    (A)   (B)   (C)

    (D)   (E)

14. The first term of a sequence is 20.
    If a term in the sequence is t and t is even, the next term is $\frac{1}{2}t$.
    If a term in the sequence is t and t is odd, the next term is $3t + 1$.
    Therefore, the first three terms in the sequence are 20, 10, 5.
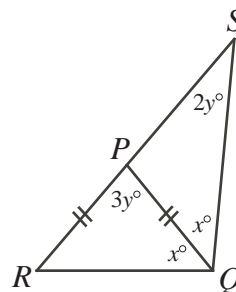   What is the 10th term of the sequence?

    (A) 2   (B) 4   (C) 5   (D) 1   (E) 8

15. If x and y are two-digit positive integers with $xy = 555$, what is $x + y$?

    (A) 52   (B) 116   (C) 66   (D) 555   (E) 45

16. In the diagram, P is on RS so that QP bisects $\angle SQR$.
Also, $PQ = PR$, $\angle RSQ = 2y°$, and $\angle RPQ = 3y°$. The
measure of $\angle RPQ$ is

(A) 90°   (B) 108°   (C) 120°

(D) 60°   (E) 72°

17. If $3 \le p \le 10$ and $12 \le q \le 21$, then the difference between the largest and smallest
possible values of $\dfrac{p}{q}$ is

(A) $\dfrac{29}{42}$ (B) $\dfrac{29}{5}$ (C) $\dfrac{19}{70}$ (D) $\dfrac{19}{12}$ (E) $\dfrac{19}{84}$

18. In the board game "Silly Bills", there are \$1, \$2 and \$3 bills.
There are 11 more \$2 bills than \$1 bills.
There are 18 fewer \$3 bills than \$1 bills.
If there is \$100 in total, then how many \$1 bills are there in the board game?

(A) 11   (B) 14   (C) 22   (D) 33   (E) 40

19. A box contains apple and pears.
An equal number of apples and pears are rotten.
$\frac{2}{3}$ of all of the apples are rotten.
$\frac{3}{4}$ of all of the the pears are rotten.
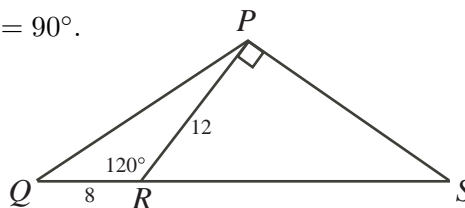What fraction of the total number of pieces of fruit in the box is rotten?

(A) $\frac{17}{24}$ (B) $\frac{7}{12}$ (C) $\frac{5}{8}$ (D) $\frac{12}{17}$ (E) $\frac{5}{7}$

20. In the diagram, R is on QS and $QR = 8$.
Also, $PR = 12$, $\angle PRQ = 120°$, and $\angle RPS = 90°$.
What is the area of $\triangle QPS$?

(A) $72\sqrt{3}$ (B) 72   (C) 36

(D) $60\sqrt{3}$ (E) $96\sqrt{3}$

Part C: Each correct answer is worth 8.

21. The circular window shown in the diagram has nine panes
of equal area. The inner circular pane has radius 20 cm
and the same centre, O, as the outer circle. The eight
lines separating the outer panes are of equal length, x cm,
and all, if extended, would pass through O. What is the
value of x, to the nearest tenth?

(A) 40.0   (B) 36.6   (C) 30.0

(D) 20.0   (E) 43.2
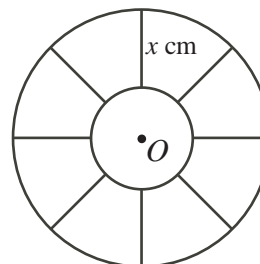
22. Suppose $N = 1 + 11 + 101 + 1001 + 10001 + \ldots + 1\overbrace{000\ldots000}^{50 \text{ zeroes}}1$.
    When N is calculated and written as a single integer, the sum of its digits is

    (A) 50   (B) 99   (C) 55   (D) 58   (E) 103

23. If x and y are integers with $(y-1)^{x+y} = 4^3$, then the number of possible values for x is

    (A) 8   (B) 3   (C) 4   (D) 5   (E) 6

24. A cube has edges of length 1 cm and has a dot marked in the centre of the top face. The cube is sitting on a flat table. The cube is rolled, without lifting or slipping, in one direction so that at least two of its vertices are always touching the table. The cube is rolled until the dot is again on the top face. The length, in centimetres, of the path travelled by the dot is

    (A) π (B) 2π (C) $\sqrt{2}\pi$ (D) $\sqrt{5}\pi$ (E) $\left(\dfrac{1+\sqrt{5}}{2}\right)\pi$

25. The average value of $(a-b)^2 + (b-c)^2 + (c-d)^2 + (d-e)^2 + (e-f)^2 + (f-g)^2$ over all possible arrangements (a, b, c, d, e, f, g) of the seven numbers 1, 2, 3, 11, 12, 13, 14 is

    (A) 398   (B) 400   (C) 396   (D) 392   (E) 394

Canadian Mathematics Competition

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